Relativistic Nucleus-Nucleus Collisions without Hydrodynamics

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The partition function of nonequilibrium distribution which we recently obtained [arXiv:0802.0259] in the framework of the maximum isotropization model (MIM) is exploited to extract physical information from experimental data on the proton rapidity and transverse mass distributions. We propose to partition all interacting nucleons into ensembles in accordance with the number of collisions. We analyze experimental rapidity distribution and get the number of particles in every collision ensemble. We argue that even a large number of effective nucleon collisions cannot lead to thermalization of nucleon system; the thermal source which describes the proton distribution in central rapidity region arises as a result of fast thermalization of the parton degrees of freedom. The obtained number of nucleons which corresponds to the thermal contribution is treated as a "nucleon power" of the created quark-gluon plasma in a particular experiment.

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The main goal of the investigations of the collisions of relativistic nuclei is extraction of a physical information about nuclear matter and its constituents. It is a matter of fact that we can get know more about quarks and gluons (constituents) just under extreme conditions, i.e. at high densities and temperatures. During last two decades one of the celebrated tools on the way of the theoretical investigations of this subject was relativistic hydrodynamics (RHD) which started to be applied to elementary particle physics from the famous Landau's paper [1].

Applying RHD one can partially describe experimental data and get know that the matter created in relativistic nucleus collisions (RNC) can be regarded on some stage of evolution as a continues one, i.e. as a liquid. Moreover, as was discovered in BNL, it can be regarded even as a perfect fluid [2] which consistent with a description of the created quark gluon plasma (QGP). The main physical quantities which can be extracted from experimental data exploiting the RHD approach are the collective (hydrodynamical) velocity and the elliptic flow parameter $v2$ of the fireball expansion. Unfortunately, all other physical information is hidden in sophisticated numerical codes which solve Euler hydrodynamic equations of motion.

In the present letter we propose approach to description of relativistic heavy-ion collisions which allows to extract the physical information from experimental data on the basis of transparent analytical model.

Maximum Isotropization Model. We separate all amount of registered nucleons into groups (ensembles) in accordance with a number of collisions, M, which every nucleon from a particular ensemble has went through. The first collision ensemble is created by the nucleons which take part just in one collision only, then $M = 1$, the second ensemble is created by the nucleons which take part in two collisions only, $M = 2$, and so on. Every ensemble contributes to momentum single-particle distribution function which can be written as

$$\frac{d^3N}{dp^3} = \sum_{M=M_{\rm min}}^{M_{\rm max}} C(M)\, D_M(\boldsymbol{p}) + C_{\rm therm}\, D_{\rm therm}(\boldsymbol{p})\,, \quad (1)$$

where the coefficients $C(M)$ and $C_{\rm therm}$ reflect the weight of the partial contribution from every M-th collision ensemble and thermal distribution, respectively, to the three-dimensional momentum spectrum, $M = 0$ corresponds to spectator particles which are not taking into account in this distribution. For the sake of simplicity we consider collision of the identical nuclei. The expression for the partial distribution functions $D_M(\boldsymbol{p})$ was derived in Ref. [3], in the c.m.s. of colliding nuclei it reads

$$D_M(\boldsymbol{p}) = \frac{1}{2z_M(\beta)}\, e^{-\beta\omega_p - \alpha \boldsymbol{p}_\perp^2} \times \left[e^{-\alpha(p_z-k_{0z})^2} + e^{-\alpha(p_z+k_{0z})^2} \right]\,, \quad (2)$$

where in the Cartesian coordinate system $\alpha \equiv 3/2Mp_{\rm max}^2$, and in the spheric system $\alpha \equiv 5/2Mp_{\rm max}^2$,

$$z_M(\beta) = \int d^3k\, e^{-\beta\omega_k - \alpha\left[\boldsymbol{k}_\perp^2 + (k_z-k_{0z})^2\right]}\,, \quad (3)$$

$\beta = 1/T$ is the inverse temperature, $\omega_p = \sqrt{m^2+\boldsymbol{p}^2}$, $\boldsymbol{p}_\perp = (p_x, p_y)$ and $0z$ is the collision axis. It is understood from (2) and (3) that the quantity $z_M(\beta)$ plays the role of the canonical single-particle partition function of the M-th collision ensemble. In some sense particular collision ensemble M can be regarded as a many-particle system frozen at some stage of evolution on the way to thermal equilibrium ($M \propto$ time).

The thermal distribution reads, $D_{\rm therm}(\boldsymbol{p}) = \exp\left[-\beta\omega_p\right]/z_{\rm therm}(\beta)$ with $z_{\rm therm}(\beta) = \int d^3k\, e^{-\beta\omega_k}$. Note, we separate in (1) the thermal contribution due to its specific role. It would seem the contribution

$D_{\rm therm}(\boldsymbol{p})$ should appear in (1) as a term of the expansion with respect to partial contributions, $D_M(\boldsymbol{p})$, when the number of collisions is big enough, i.e. when $M_{\rm max} \to \infty$. Meanwhile, because of finite life time of the system (fireball) and hence finite number of elastic and inelastic collisions of nucleons this limit regime of hadron dynamics, $M \to \infty$, is not achieved and $M_{\rm max}$ is finite. However, we include in (2) a thermal source because it has another nature. We will return later to the discussion of this matter.

There are two additional quantities in (2)-(3), k_{0z} and $p_{\rm max}$, which are the external parameters determined by the particular experimental conditions. The values $\pm k_{0z}$ are the initial momenta of nucleons in c.m.s. Indeed, due to the specifics of heavy-ion collisions we know exactly the initial momenta of the nucleons in both colliding nuclei. Two Gaussians in the brackets on the r.h.s. of (2) reflect a smearing around initial momenta which is due to collisions of nucleons and were obtained with the help of the saddle-point approximation. Note, for $M = 1, 2, 3$ this approximation is not used.

Under the notion "collision" we mean elastic rescattering as well as inelastic scattering (reactions), which include a creation of secondary particles. In the transverse direction both nuclei have the same zero initial momentum. Then, for both nuclei the smearing around this value is appeared in (2) as the common factor, $\exp\left[-\alpha \boldsymbol{p}_\perp^2\right]$. The covariance of the Gaussian depends on the number of collisions M and the maximally allowed transferred momentum.

The rapidity distribution was obtained after integration of (1) with respect to the nucleon transverse mass, $m_\perp = (m_N^2 + \boldsymbol{p}_\perp^2)^{1/2}$, where m_N is the nucleon mass and rapidity, y, is defined as $\tanh y = p_z/\omega_p$. With respect to new variables one gets, $d^3p = d\phi\, \omega_p\, m_\perp\, dm_\perp\, dy$.

As a first step of our approach we consider a central collision of identical nuclei and we assume an azimuth symmetry of radiation of the particles. Rapidity spectrum of registered particles looks like

$$\frac{dN}{dy} = \sum_{M=M_{\rm min}}^{M_{\rm max}} C(M)\, \varphi_M(y) + C_{\rm therm}\, \varphi_{\rm therm}(y)\,, \quad (4)$$

where

$$\varphi_M(y) = 2\pi \cosh y \int_{m_N}^{\infty} dm_\perp\, m_\perp^2\, D_M(m_\perp, y)\,. \quad (5)$$

To obtain $\varphi_{\rm therm}(y)$ we put $D_{\rm therm}$ in place of D_M on the r.h.s. of (5). Double differential spectrum which depends on the transverse mass is obtained from (1)

$$\frac{d^2N}{m_\perp dm_\perp dy} = 2\pi\, m_\perp \cosh y \Big[\sum_M C(M) D_M(m_\perp, y) + C_{\rm therm}\, D_{\rm therm}(m_\perp, y) \Big]\,. \quad (6)$$

Usually the mode $M = 1$ does not give contribution to the particular experimental rapidity window. In this case we can set $C(1) \simeq 0$ and start summation in (4) from $M_{\rm min} = 2$.

Extraction of the physical information from experimental data. With making use of the rapidity distribution (4) we fit the experimental data on the rapidity distribution of net protons which were measured at the CERN SPS (NA49 Collaboration) [4]. The slope parameter β was first extracted from double differential yield for protons with the use of the thermal distribution. The proton data is remarkable in that sense that we know exactly the initial momentum, k_{0z}, of every nucleon. The fit was carried out with a help of the program MINUIT, variable parameters are coefficients $C(M)$ and $C_{\rm therm}$ and parameter, which confines the momentum space, $p_{\rm max}$. The values of the obtained parameters for $T = 1/\beta = 248$ MeV are shown in Table 1. All evaluations are carried out in the c.m.s. of colliding nuclei with use of the spheric coordinate system.

Table 1

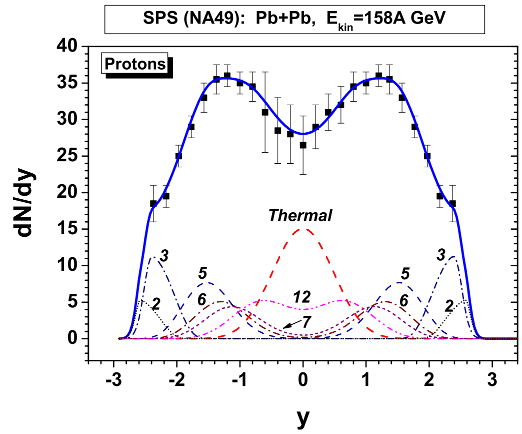
$C(2)$	$C(3)$	$C(4)$	$C(5)$	$C(6)$	$C(7)$	$C(8)$
4.47	11.9	28.9	11.7	10.5	9.5	9.57

$C(9)$	$C(10)$	$C(11)$	$C(12)$	$C_{\rm therm}$	$p_{\rm max}$ (GeV/c)
10.0	10.6	11.2	11.8	18.04	1.275

The obtained theoretical curves together with experimental data are depicted in Fig. 1. Broken curves (see upper panel) marked by the numbers M and solid thick curve (blue in on-line presentation) represent the partial contributions from every ensemble, $C(M)\cdot\varphi_M(y)$, and complete theoretical proton rapidity distribution, respectively. The thermal contribution is represented by central bell-like dashed curve (red in on-line presentation).

The integral on the r.h.s. of Eq. (3) which gives rise to single-particle partition function $z_M(\beta)$ is defined in the rapidity range $[-Y_{\rm cm}, Y_{\rm cm}]$, where $Y_{\rm cm} = Y_{\rm beam}/2$. Then, the functions $\varphi_M(y)$ are normalized to unity in the same range. If one integrates Eq. (4) with respect to rapidity in this range it is easy to find that a result of integration on the r.h.s. equals to the sum of all coefficients $C(M)$ plus $C_{\rm therm}$. At the same time the value of this integral equals the area under the "rapidity" curve (solid, thick blue curve) in Fig.1 or to the total number of participated protons which would be registered in case if the total rapidity window $[-Y_{\rm cm}, Y_{\rm cm}]$ is allowed experimentally: $N_p^{(\rm tot)} = \sum_M C(M) + C_{\rm therm}$. For instance, for NA49 experimental data [4], we obtain $N_p^{(\rm tot)} = 151$.

So, every coefficient $C(M)$ tells us how many protons undergo M effective collisions or what is the popularity of every collision ensemble. For instance, ensemble of the protons which participated in nine effective collisions, $M = 9$, consists of 10 protons, i.e. $C(9) \simeq 10$. What is very important, we learn from



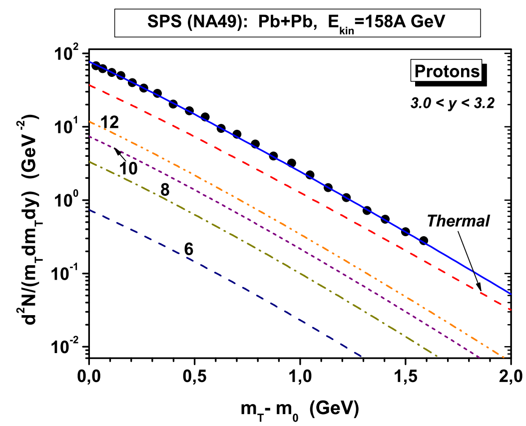


FIG. 1: *Upper panel:* The result of the fit (thick solid curve, blue in on-line presentation) to experimental data [4] on rapidity distribution (see Eq. (4)). Broken curves marked by the numbers M represent the partial contributions from every collision ensemble, $C(M) \cdot \varphi_M(y)$, the thermal contribution is represented by central Gaussian-like dashed curve (red in on-line presentation).
Lower panel: The thick solid curve (blue in on-line presentation) represents the evaluation of the $m_\perp$-spectrum obtained in accordance with Eq. (6) were we use the same values of the coefficients $C(M)$ which were obtained as a result of the fit of the dN/dy data. Experimental data points are from [4]. Broken curves marked by the numbers M represent the partial contributions from every collision ensemble, $2\pi\, m_\perp \cosh y \cdot C(M) D_M(m_\perp, y)$.

this expansion that $C_{\rm therm} \simeq 18$, it means that approximately eighteen protons come from a thermal source what makes up 12% (twelve percent) of all participated protons.

Now we would like to draw attention to the ensemble with maximal collisions, $M = 12$. This value was determined from UrQMD [5] evaluation of the mean maximum number of the effective nucleon collisions. It turns out, as it is seen from Fig.1 (upper panel), that the partial function $\varphi_{12}(y)$ does not "fill in" successfully the central rapidity region. That is why the presence of the thermal function $\varphi_{\rm therm}(y)$ (Gaussian like curve in the center of Fig.1) is necessary in the expansion (4). Even this big number of effective collisions, $M_{\rm max} = 12$, cannot give rise to a source which is compared in the central rapidity region with a thermal one. It is the main reason why the thermal source should be presented in the expansion (4).

Note, in the case of finite and small number of experimental points the set of functions, $\varphi_M(y)$, is overcomplete. To choose a unique configuration of the variable parameters we use the maximum entropy method [7].

In this analysis we are coming to one of the main conclusions, which can be derived from our model: The thermal source has absolutely different nature of origination, it cannot be created just due to the hadron reactions of nucleons which result in randomization and subsequent isotropization of the nucleon momentum. The thermal source can emerge as a result of appearance "at once" of many new degrees of freedom. We know just one candidate to this role, it is the quark-gluon plasma, for instance, its creation can occur in collision of nucleons in the presence of a dense medium, $N + N \to n_g + n_q$. Then, a many-parton system, which emerges in the collision, consists of $n_g \gg 1$ gluons and $n_q \gg 1$ quarks. All momenta of quarks and gluons can be regarded from the very beginning as random ones and thermalization of the system occur during a time span $\tau_{\rm therm} = 0.6$ fm/c [8]. Hence, the protons which come from the thermal source indicate the presence of the QGP in the fireball and we can determine a power of the QGP by the number of protons outcoming from the thermal source or by the value of $C_{\rm therm}$.

Actually, the total number of nucleons which appear as a result of hadronization of the QGP can be then evaluated with accounting for isotope composition of the colliding nuclei: $N_N^{(\rm QGP)} = C_{\rm therm} \dfrac{A}{Z}$. For instance, in the experiment under consideration we find $C_{\rm therm} \simeq 18$, and then $N_N^{(\rm QGP)} \simeq 46$, i.e. approximately 46 nucleons were created by the QGP or by several QGP drops. This makes up 12% from a total number of net nucleons which are the participants of the collision, $N_N^{(\rm participants)} \simeq 382$. Then, we estimate a "nucleon power" of the QGP, $P_{\rm qgp} \equiv N_N^{(\rm QGP)}/N_N^{(\rm participants)}$, which was created in nucleus-nucleus collision. For instance, it turns out that $P_{\rm qgp} \simeq 12\%$ in Pb+Pb collisions (SPS) at $E_{\rm kin} =$ 158A GeV.

The same analysis was carried out for proton distribution from 11.6A GeV/c Au + Au collisions measured by the E802 Collaboration [6]. The fit to the experimental data (0-3% centrality) allows to extract the values of parameters which are shown in Table 2. For this experiment UrQMD [5] evaluation gives

the mean maximum number of the effective nucleon collisions $M_{\max} = 13$. The width of the rapidity window in this experiment avoids the necessity to take into account the collision ensemble $M = 1$ too.

Table 2

$C(2)$	$C(3)$	$C(4)$	$C(5)$	$C(6)$	$C(7)$	$C(8)$	$C(9)$
12.2	14.5	6.5	5.1	5.1	5.8	6.8	8.2

$C(10)$	$C(11)$	$C(12)$	$C(13)$	C_{therm}	$p_{\max}$ (GeV/c)
9.7	11.4	13.0	14.7	37.9	0.724

Theoretical curves and experimental data are depicted in Fig. 2. Notations and marks have the same meaning as in the previous figure. In the lower panel the solid curves represent the evaluation of the $m_\perp$-spectra in different rapidity windows. Remind, these curves are obtained without additional fitting of the data. We just use the values of parameters from the Table 2. Meanwhile, on the first step, before fitting the rapidity distribution, we estimate with the help of the thermal distribution the slope parameter β and find $T = 280$ MeV.

We can estimate as well the nucleon power of the produced QGP in experiment Au + Au at 11.6A GeV/c (0-3% centrality). In this case $C_{\text{therm}} \simeq 38$ and $N_p^{(\text{tot})} \simeq 151$, hence we obtain $P_{\text{qgp}} \simeq 25\%$.

Summary and discussion. In the proposed Maximum Isotropization Model [3] the maximum number of collisions (reactions), $M_{\max}$, assumed to be finite and determined by the nuclear number A, initial energy and centrality. With the help of the UrQMD transport model [5] it was found that for SPS (Pb+Pb, 158A GeV) conditions [4], $M_{\max} = 12$, and for AGS (Au+Au, 11.6A GeV/c, 0-3% centrality) conditions [6], $M_{\max} = 13$. Utilizing thermal distribution we extract a slope parameter from experimental data on the proton $m_\perp$-spectra: for SPS conditions [4], $T = 248$ MeV, and for AGS conditions [6] (0-3% centrality), $T = 280$ MeV. It is evidently seen from Fig. 1 (lower panel) that the $m_\perp$-spectrum is mainly determined by the thermal component, and in any case the slope of the partial contribution, marked by M, is approximately the same as of thermal distribution. Exactly of that reason the $m_\perp$-spectrum is low informative about collision ensembles or the information about rescattered nucleons almost lost in this presentation. On the other hand, the $m_\perp$-spectrum as a trigger gives possibility to extract a value of the slope parameter.

Next, we made the fit of experimental data [4, 6] on the rapidity distribution of the net protons and obtained the set of coefficients $C(M)$ (see Tables 1, 2) which are nothing more as an absolute number of protons in every collision ensemble. Note, the proton data is interesting first of all because we know an exact value of the initial nucleon momentum. As a matter of fact, the partial expansion, $dN/dy = \sum_{M=M_{\min}}^{M_{\max}} C(M)\,\varphi_M(y)$ (see (4)), where we use obtained coefficients $C(M)$ from Tables 1, 2, makes a good description of the experimental data on rapidity distribution, except the central rapidity region. It means that the central rapidity region cannot be described just by finite number of nucleon rescatterings (hadron reactions). Then, we are forced to take into account also the thermal source, which evidently has a different nature. We assume that this source is a thermalized multi-parton system (QGP) [8] which through hadronization process emits totally thermalized nucleons. The knowledge of the number of protons, C_{therm}, which come from the QGP, gives us

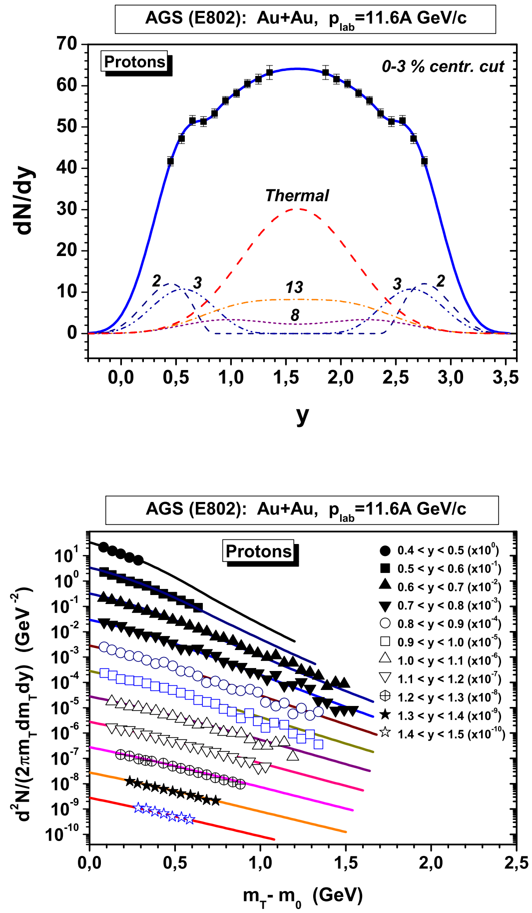


FIG. 2: *Upper panel:* The result of the fit (thick solid curve, blue in on-line presentation) to experimental data [6] on rapidity distribution (see Eq. (4)). Broken curves marked by the numbers M represent the partial contributions from every collision ensemble, $C(M) \cdot \varphi_M(y)$, the thermal contribution is represented by central Gaussian-like dashed curve (red in on-line presentation).
Lower panel: The solid curves represent the evaluation of the $m_\perp$-spectra obtained in accordance with Eq. (6) were we use the same values of the coefficients $C(M)$ which were obtained as a result of the fit of the dN/dy data. Experimental data points are from [6].

a possibility to evaluate the "nucleon power" of the QGP, $P_{\rm qgp}$, created in the particular experiment on nucleus-nucleus collision. We find that for SPS conditions [4], $P_{\rm qgp} \simeq 12\%$, and for AGS conditions [6] (0-3% centrality), $P_{\rm qgp} \approx 25\%$. So, following the proposed criterium we can claim that QGP (as a nucleon source) was created not only at SPS energies [9] but it was also created, even more powerful with respect to nucleons, in the central collisions at AGS energies. Meanwhile, in accordance with UrQMD estimations the number of pions created in hadron reactions at the SPS [4] and AGS [6] energies are approximately the same. Hence, the number of pions created by the thermal source at the SPS is much bigger than the number of pions created by the thermal source at the AGS. From that we can conclude that "pion power" of QGP created in nucleus-nucleus collisions at the SPS up to one order higher than that one created at the AGS.

All this leaves us with the continued challenge of applying the model to other experiments and problems.

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